

Can neutrino vacuum support the wormhole?

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Abstract

The renormalised vacuum expectation values of massless fermion (conventionally call it neutrino) stress-energy tensor are calculated in the static spherically-symmetrical wormhole topology. Consider the case when the derivatives of metric tensor over radial distance are sufficiently small (in the scale of radius) to justify studying a few first orders of quasiclassical (WKB) expansion over derivatives. Then we find that violation of the averaged weak energy condition takes place irrespectively of the detailed form of metric. This is a necessary condition for the neutrino vacuum to be able to support the wormhole geometry. In this respect, neutrino vacuum behaves like electromagnetic one and differs from the conformal scalar vacuum which does not seem to violate energy conditions for slowly varied metric *automatically*, but requires self-consistent wormhole solution for this.

1.Introduction. The possibility of existence of static spherically-symmetrical traversible wormhole as topology-nontrivial solution to the Einstein equations has been first studied by Morris and Thorne in 1988 [1]. They have found that the material which threads the wormhole should violate weak energy condition at the throat of the wormhole, that is, radial pressure should exceed the density. Moreover, Morris, Thorne and Yurtsever [2] have pointed out that averaged weak energy condition (i.e. that integrated over the radial direction) should also be violated. Since that time much activity has been developed in studying the wormhole subject (see, e.g., review by Visser [3]) of which we consider here the possibility of existence of self-consistent wormhole solutions to semiclassical Einstein equations. Checking this possibility requires finding vacuum expectation value of the stress-energy tensor as functional of geometry and solving the Einstein equations with this tensor as a source.

Recently self-consistent spherically symmetric wormhole solution has been found numerically by Hochberg, Popov and Sushkov [4] for the quantised scalar field vacuum playing the role of a source for the gravitation. These authors employ vacuum expectation values of the stress-energy tensor for the scalar field found by Anderson, Hiscock and Samuel [5] although Anderson, Hiscock and Taylor have argued (without solving the backreaction problem, however) that these values for massive minimally and/or conformally coupled scalars fail to meet the requirements for maintaining five particular types of static spherically symmetric wormholes [6]. As for the experimentally known fields, we have calculated in [7] the renormalised stress-energy tensor of electromagnetic vacuum with the help of the covariant geodesic point separation method of regularisation [8]. It has been found to violate weak energy condition at the wormhole throat in the first nonvanishing order in the expansion over the derivatives of metric (the WKB expansion). This is a necessary condition of existence of self-maintained wormhole solution. Important is that this violation takes place irrespectively of the detailed form of metric.

Also violation of the averaged weak energy condition is necessary (but not sufficient, of course) condition for the existence of vacuum self-maintained wormhole. The validity of the weak averaged energy condition for the self-consistent solution to the Einstein equations has been studied by Flanagan and Wald [9] for the massless scalar field. Flanagan and Wald have found that averaged weak energy condition holds for self-consistent solution if being additionally averaged transverse to the geodesic using a suitable Plank scale smearing function. The Flanagan-Wald result is obtained in the context of perturbation theory about flat spacetime, i.e. for the Minkowski topology. In our preceding paper [10] dealing with electromagnetic field we note that validity of the averaged weak energy condition is substantially defined by the space(-time) topology. This condition is violated in the first nonvanishing order of WKB expansion over derivatives of metric (over radial coordinate) if topology is that of wormhole.

Important is possibility to have macroscopic wormhole size under proper conditions. Flanagan and Wald [9] and Ford and Roman [11] have argued that wormhole size would be Plank-scale. These arguments are based on the assumption that coefficient at the curvature squared (Weyl term) in the effective gravity action is of Plank scale value. Meanwhile, experimental bounds on this coefficient are not very restrictive [12], and in

[10] we speculate that this allows the wormhole size to be as large as radius of the Sun. A possible large value of the Weyl term might be provided by infra-red contribution into effective action from the massless fields, such as electromagnetic one.

In the given note we perform analogous calculations for the stress-energy of the neutrino vacuum in the wormhole topology. We find that violation of the energy conditions takes place in the self-consistent manner analogously to what occurs in the electromagnetic vacuum in the wormhole topology [7, 10]. This is the necessary condition and justifies further work towards construction of self-consistent wormhole solution in the real physical vacuum of spin 1/2 and 1 fields.

2.Calculation. The notations we employ are mainly those of ref.[13] used in ref.[8]. The notations for the metric functions $r(\rho)$, $\Phi(\rho)$ are read from the following expression for the line element:

$$ds^2 = r^2(\rho)(d\theta^2 + \sin^2\theta d\phi^2) + d\rho^2 - \exp(2\Phi)dt^2. \quad (1)$$

The μ, ν, λ, \dots denote world tensor indices t, ρ, θ, ϕ while $\alpha, \beta, \gamma, \dots$ are purely space such indices. We also need notation for the local flat space indices $a, b, c, \dots = 1, 2, 3$. In the case of spherical symmetry it is convenient to use decomposition of spinor field over spherical waves. Therefore we consider neutrino field as particular vanishing-mass case of the four-component Dirac field and take γ -matrices in the local flat Minkowski coordinates x^0, x^1, x^2, x^3 in the usual standard representation:

$$\gamma_0 = \begin{pmatrix} i & 0 \\ 0 & -i \end{pmatrix}, \quad \gamma_a = \begin{pmatrix} 0 & i\sigma_a \\ -i\sigma_a & 0 \end{pmatrix}, \quad (2)$$

σ_a being Pauli matrices, $\sigma_1\sigma_2 = i\sigma_3, \dots$. We choose tetrad by requiring that γ_μ were proportional to the expressions which would follow if x^0, x^1, x^2, x^3 were treated as usual Cartesian coordinates t, x, y, z and t, ρ, θ, ϕ were treated as usual spherical flat coordinates:

$$\begin{aligned} \gamma_t &= \gamma_0 \exp(\Phi), \\ \gamma_\rho &= \gamma_1 \sin\theta \cos\phi + \gamma_2 \sin\theta \sin\phi + \gamma_3 \cos\phi, \\ \gamma_\theta &= (\gamma_1 \cos\theta \cos\phi + \gamma_2 \cos\theta \sin\phi - \gamma_3 \sin\phi)r, \\ \gamma_\phi &= (-\gamma_1 \sin\phi + \gamma_2 \cos\phi)r \sin\theta. \end{aligned} \quad (3)$$

The σ_α are defined by analogous formulas with γ replaced by σ . It proves convenient to write bispinor spherical waves in the form

$$\begin{aligned} \psi &= \frac{\exp(-\Phi/2)}{r} \begin{pmatrix} \eta_+ \Omega_+ \\ \eta_- \Omega_- \end{pmatrix} \exp(-i\omega t), \\ \Omega_{-lm} &= \begin{pmatrix} -\sqrt{\frac{l-m}{2l+1}} Y_{lm} \\ \sqrt{\frac{l+m+1}{2l+1}} Y_{l,m+1} \end{pmatrix}, \quad \Omega_{+lm} = \begin{pmatrix} \sqrt{\frac{l+m}{2l-1}} Y_{l-1,m} \\ \sqrt{\frac{l-m-1}{2l-1}} Y_{l-1,m+1} \end{pmatrix} \end{aligned} \quad (4)$$

and analogous one with $\eta_-\Omega_-$ and $\eta_+\Omega_+$ interchanged. It is implied that spherical function $Y_{lm} = 0$ at $m > l$. Here $l \geq 1$ and the full momentum $j = l - 1/2$ with $m + 1/2$ being it's projection. The Dirac conjugate is $\bar{\psi} = \psi^\dagger \gamma^0$ and spherical spinors satisfy

$$\sigma_\rho \Omega_\pm = \pm i \Omega_\mp, \quad (\sigma^\theta \partial_\theta + \sigma^\phi \partial_\phi) \Omega_\pm = -i \frac{l \mp 1}{r} \Omega_\mp, \quad \sum_m \Omega_\pm^\dagger \Omega_\pm = \frac{l}{2\pi}. \quad (5)$$

The covariant derivative of a spinor $\psi_{;\mu} \equiv D_\mu \psi = \partial_\mu \psi - \Gamma_\mu \psi$ can be defined from the requirement

$$\gamma_{\mu;\nu} \equiv \gamma_{\mu,\nu} - \Gamma_{\mu\nu}^\lambda \gamma_\lambda - \Gamma_\nu \gamma_\mu + \gamma_\mu \Gamma_\nu = 0. \quad (6)$$

This gives

$$D_t = \partial_t + \frac{1}{2} \Phi' \gamma_t \gamma_\rho, \quad D_\rho = \partial_\rho, \quad D_A = \partial_A + \frac{r' - 1}{2r} \gamma_A \gamma_\rho, \quad A = \theta, \phi. \quad (7)$$

Description of the formalism is completed by setting the standard expression for the action

$$S = \frac{i}{4} \int (\bar{\psi} \gamma^\mu \psi_{;\mu} - \overline{\psi_{;\mu}} \gamma^\mu \psi) \sqrt{-g} d^4 x \quad (8)$$

which provides the stress-energy

$$T_{\mu\nu} = -\frac{i}{4} (\bar{\psi} \gamma_\mu \psi_{;\nu} + \overline{\psi_{;\nu}} \gamma_\mu \psi). \quad (9)$$

The equation of motion $\gamma^\mu \psi_{;\mu} = 0$ in terms of the two-component function $\eta = \begin{pmatrix} \eta_+ \\ \eta_- \end{pmatrix}$ takes the form (and the same for ψ with $\eta_-\Omega_-$ and $\eta_+\Omega_+$ interchanged):

$$\omega \begin{pmatrix} \eta_+ \\ \eta_- \end{pmatrix} = \frac{d}{dz} \begin{pmatrix} \eta_- \\ -\eta_+ \end{pmatrix} + lU \begin{pmatrix} \eta_- \\ \eta_+ \end{pmatrix} \equiv \mathcal{O} \begin{pmatrix} \eta_+ \\ \eta_- \end{pmatrix} \quad (10)$$

where we have denoted $U = \exp(\Phi)/r$ and introduced the new variable z via $dz = \exp(-\Phi) d\rho$.

Symmetrical separation of the points x, \tilde{x} according to the Christensen's prescription [8] for the regularised form of the stress-energy tensor gives, e.g.,

$$T_{tt}^{\text{reg}} = -\frac{i}{4} [\bar{\psi} \gamma_t (\tilde{\psi}_{;t})_x + \overline{(\tilde{\psi})_x} \gamma_t \psi_{;t} - \overline{\psi_{;t}} \gamma_t (\tilde{\psi})_x - \overline{(\tilde{\psi}_{;t})_x} \gamma_t \psi]. \quad (11)$$

Here $(\tilde{\psi})_x, (\tilde{\psi}_{;t})_x$ are the fields $\psi(\tilde{x}), \psi_{;t}(\tilde{x})$, respectively, transported in parallel way from \tilde{x} to x along the geodesic. We split the point in radial direction so that $x = (t, \rho, \theta, \phi)$, $\tilde{x} = (t, \tilde{\rho}, \theta, \phi)$, $\tilde{\rho} - \rho = \epsilon \rightarrow 0$.

Now we substitute ψ as the field operator expanded in terms of creation and annihilation operators into (11). As in [8], the operator ordering $\psi^+ \dots \psi \Rightarrow \frac{1}{2}(\psi^+ \dots \psi - \psi \dots \psi^+)$ is implied. The vacuum expectation values of $T_{\mu\nu}^{\text{reg}}$ will be denoted as components themselves: this will not lead to any confusion. These values are

$$\begin{pmatrix} T_t^t \\ T_\rho^\rho \\ T_\theta^\theta \end{pmatrix}^{\text{reg}} = \frac{1}{4\pi r \tilde{r}} \exp\left(-\frac{\Phi}{2} - \frac{\tilde{\Phi}}{2}\right) \left\{ \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix} \left[(\exp(-\Phi) + \exp(-\tilde{\Phi})) S_1 - \frac{\tilde{\Phi}' - \Phi'}{2} S_a \right] + \begin{pmatrix} 0 \\ 1 \\ -\frac{1}{2} \end{pmatrix} \left[\left(\frac{1}{r} + \frac{1}{\tilde{r}}\right) S_s + \left(\frac{\tilde{r}'}{\tilde{r}} - \frac{r'}{r}\right) S_a \right] \right\}. \quad (12)$$

The tilde denotes the function at $\tilde{\rho}$. Here we introduce the three basic sums over quantum numbers n (which labels the solutions to the eq.(10)) and l . (The summation over m is made in closed form thus eliminating the angle θ, ϕ dependence and leading to $T_\phi^\phi = T_\theta^\theta$). These sums are

$$\begin{aligned} S_1 &= \sum_{n,l} l \omega \theta(\omega) (\eta_+ \tilde{\eta}_+ + \eta_- \tilde{\eta}_-), \\ S_s &= \sum_{n,l} l^2 \theta(\omega) (\eta_+ \tilde{\eta}_- + \eta_- \tilde{\eta}_+), \\ S_a &= \sum_{n,l} l \theta(\omega) (\eta_+ \tilde{\eta}_- - \eta_- \tilde{\eta}_+). \end{aligned} \quad (13)$$

Here indices n, l at η_\pm and ω are suppressed, $\theta(\omega) = \frac{1}{2} \left(1 + \frac{\omega}{|\omega|}\right)$, and η is normalised as

$$\int_{-\infty}^{+\infty} (\eta_+^2 + \eta_-^2) dz = 1 \quad (14)$$

(we use real functions η_\pm).

The operator \mathcal{O} in (10) is Hermitian w.r.t. the internal product like (14) and can be made self-adjoint by imposing appropriate boundary conditions at $z = \pm M$; then we put $M \rightarrow \infty$. The two-component eigenvector η of \mathcal{O} form the orthogonal set and by the completeness property

$$\sum_n \eta(z) \eta^+(\tilde{z}) = \mathbf{1} \cdot \delta(z - \tilde{z}) \quad (15)$$

(here $\mathbf{1}$ is unit 2×2 matrix). Thereby summation over n in the stress-energy is reduced to finding the kernel of the operator functions $f(\mathcal{O}) = \mathcal{O} \theta(\mathcal{O})$ or $\theta(\mathcal{O})$:

$$\sum_n f(\omega) \eta(z) \eta^+(\tilde{z}) = f(\mathcal{O}) \delta(z - \tilde{z}). \quad (16)$$

These functions can be rewritten as

$$\theta(\mathcal{O}) = \frac{1}{2} \left(1 + \frac{\mathcal{O}}{\sqrt{\mathcal{O}^2}}\right), \quad \mathcal{O} \theta(\mathcal{O}) = \frac{1}{2} (\mathcal{O} + \sqrt{\mathcal{O}^2}). \quad (17)$$

Nonlocality upon the action on $\delta(z - \tilde{z})$ arises from the terms with $\sqrt{\mathcal{O}^2}$. Corresponding result at $\tilde{z} \neq z$ can be expressed in terms of Green function $G(s, l; z, \tilde{z})$ which satisfies

$$-G''_{zz} + \omega^2(z)G = \delta(z - \tilde{z}), \quad \omega^2(z) \equiv s^2 + l^2 U^2 + lU'_z \sigma_3 \quad (18)$$

in the following way:

$$\begin{aligned} \sum_n \eta(z) \eta^+(\tilde{z}) \omega \theta(\omega) &= - \int_0^\infty \frac{s^2 ds}{\pi} G(s, l; z, \tilde{z}), \\ \sum_n \eta(z) \eta^+(\tilde{z}) \theta(\omega) &= \left(i\sigma_2 \frac{d}{dz} + lU\sigma_1 \right) \int_0^\infty \frac{ds}{\pi} G(s, l; z, \tilde{z}). \end{aligned} \quad (19)$$

Since the matrix $\omega^2(z)$ is diagonal, the WKB expansion for G over the powers of $\Delta z = \tilde{z} - z$ and over the derivatives of $\omega^2(z)$ does not differ from that obtained for c -number $\omega^2(z)$ in our previous paper [10]:

$$\begin{aligned} 2G(z, \tilde{z}) &= \exp(-\omega \Delta z) \left\{ \frac{1}{\omega} - \frac{1}{8} \frac{(\omega^2)''}{\omega^5} + \frac{5}{32} \frac{(\omega^2)'^2}{\omega^7} \right. \\ &\quad + \Delta z \left[-\frac{1}{4} \frac{(\omega^2)'}{\omega^3} - \frac{1}{8} \frac{(\omega^2)''}{\omega^4} + \frac{5}{32} \frac{(\omega^2)'^2}{\omega^6} \right] \\ &\quad + \Delta z^2 \left[-\frac{1}{4} \frac{(\omega^2)'}{\omega^2} - \frac{1}{8} \frac{(\omega^2)''}{\omega^3} + \frac{5}{32} \frac{(\omega^2)'^2}{\omega^5} \right] \\ &\quad \left. + \Delta z^3 \left[-\frac{1}{12} \frac{(\omega^2)''}{\omega^2} + \frac{5}{48} \frac{(\omega^2)'^2}{\omega^4} \right] + \Delta z^4 \frac{1}{32} \frac{(\omega^2)'^2}{\omega^3} \right\}. \end{aligned} \quad (20)$$

Here the derivatives are over z . One need to additionally expand over U'_z entering $\omega^2(z)$.

Substitute this into the expression for the stress-energy and introduce instead of s a new integration variable $q = \Delta z \sqrt{s^2 l^{-2} + U^2}$. Thereby we obtain a collection of terms containing powers of derivatives of U over z (that is, of r and Φ over ρ) times coefficients of the type

$$\Delta z^k \int_{U\Delta z}^\infty \frac{dq}{q^p \sqrt{q^2 - U^2 \Delta z^2}} \frac{d^j}{dq^j} \frac{f(q)}{q^2} \quad (21)$$

where

$$f(q) \equiv q^2 \sum_{l=1}^\infty l \exp(-ql) = \frac{q^2}{4 \sinh^2 \frac{q}{2}}. \quad (22)$$

Expansion of the sums (13) in powers of Δz is achieved by carefully expanding $f(q)$ in Taylor series around $q = 0$.

When substituting S_1, S_s, S_a into (12) one should also expand \tilde{r} and $\tilde{\Phi}$ over Δz and express Δz in terms of $\epsilon = \Delta \rho$. The result is (the derivatives are over ρ now):

$$8\pi^2 r^4 T_t^{t,\text{reg}} = -8\frac{r^4}{\epsilon^4} + \frac{1}{3}\frac{r^2}{\epsilon^2} (1 - r'^2 + 2r''r) - \frac{1}{3}\frac{r}{\epsilon}r' + \frac{1}{60}\ln\frac{Lr}{\epsilon} + \frac{23}{72}r'^2 - \frac{5}{36}r''r, \quad (23)$$

$$8\pi^2 r^4 T_\rho^{\rho,\text{reg}} = +24\frac{r^4}{\epsilon^4} - \frac{1}{3}\frac{r^2}{\epsilon^2} \left[1 + r^2 \left(2\Phi'' + 2\Phi'^2 - 2\Phi'\frac{r'}{r} - \frac{r'^2}{r^2} + 4\frac{r''}{r} \right) \right] + \frac{1}{3}\frac{r}{\epsilon}r' \\ + \frac{1}{60} \left(\ln\frac{Lr}{\epsilon} - 1 \right) + \frac{1}{72}r^2 \left(-2\Phi'' - 2\Phi'^2 + 2\Phi'\frac{r'}{r} - 13\frac{r'^2}{r^2} \right), \quad (24)$$

$$8\pi^2 r^4 T_\theta^{\theta,\text{reg}} = -8\frac{r^4}{\epsilon^4} + \frac{1}{3}\frac{r^2}{\epsilon^2} \left(\Phi'' + \Phi'^2 - \Phi'\frac{r'}{r} + \frac{r''}{r} \right) - \frac{1}{60} \left(\ln\frac{Lr}{\epsilon} - \frac{1}{2} \right) \\ + \frac{1}{72}r^2 \left(\Phi'' + \Phi'^2 - \Phi'\frac{r'}{r} - 5\frac{r'^2}{r^2} + 5\frac{r''}{r} \right). \quad (25)$$

The divergences at $\epsilon \rightarrow 0$ can be eliminated by subtracting $T_\nu^{\mu,\text{div}}$, the stress-energy corresponding to the divergent part of the effective action derived by Christensen [8] (simultaneously the cosmological constant, Einstein gravity constant and coefficient at the Weyl tensor squared $C_{\mu\nu\lambda\rho}C^{\mu\nu\lambda\rho}$ in the effective action are set equal to their experimental values). Substituting our metric into the Christensen's formula for the spinor field gives:

$$8\pi^2 r^4 T_t^{t,\text{div}} = -8\frac{r^4}{\epsilon^4} + \frac{1}{3}\frac{r^2}{\epsilon^2} (1 - r'^2 + 2r''r) + \frac{1}{60}\ln\frac{\Lambda}{\epsilon} + \frac{1}{3}r'^2 - \frac{1}{6}r''r, \quad (26)$$

$$8\pi^2 r^4 T_\rho^{\rho,\text{div}} = +24\frac{r^4}{\epsilon^4} - \frac{1}{3}\frac{r^2}{\epsilon^2} \left[1 + r^2 \left(2\Phi'' + 2\Phi'^2 - 2\Phi'\frac{r'}{r} - \frac{r'^2}{r^2} + 4\frac{r''}{r} \right) \right] + \\ + \frac{1}{60} \left(\ln\frac{\Lambda}{\epsilon} - 1 \right) - \frac{1}{36}r^2 \left(\Phi'' + \Phi'^2 + 6\frac{r'^2}{r^2} \right), \quad (27)$$

$$8\pi^2 r^4 T_\theta^{\theta,\text{div}} = -8\frac{r^4}{\epsilon^4} + \frac{1}{3}\frac{r^2}{\epsilon^2} \left(\Phi'' + \Phi'^2 - \Phi'\frac{r'}{r} + \frac{r''}{r} \right) - \frac{1}{60}\ln\frac{\Lambda}{\epsilon} - \frac{1}{12}r'^2 + \frac{1}{12}r''r. \quad (28)$$

The coefficient at the Weyl term is the only one which is both logarithmically UV and (in the considered case of zero mass) IR divergent, and Λ is the IR cut-off. The Christensen's procedure includes also forming half of the sum of the components $T_{\mu\nu}$ corresponding to point separations $\epsilon = \pm|\epsilon|$ (above formulas are given for $\epsilon > 0$), thereby only even powers of $|\epsilon|$ are left (and also $|\epsilon|$ rather than ϵ enters the logarithm). Subtracting $T_{\mu\nu}^{\text{div}}$ from $T_{\mu\nu}^{\text{reg}}$ cancels the divergences at $\epsilon \rightarrow 0$ (this is a useful check of our calculation), and we finally obtain

$$8\pi^2 r^4 T_t^{t,\text{ren}} = +\frac{1}{60}\ln\frac{Lr}{\Lambda} - \frac{1}{72}r'^2 + \frac{1}{36}r''r, \quad (29)$$

$$8\pi^2 r^4 T_\rho^{\rho,\text{ren}} = +\frac{1}{60}\ln\frac{Lr}{\Lambda} + \frac{1}{36}\Phi'r'r - \frac{1}{72}r'^2, \quad (30)$$

$$8\pi^2 r^4 T_\theta^{\theta,\text{ren}} = -\frac{1}{60}\ln\frac{Lr}{\Lambda} + \frac{1}{120} + \frac{1}{72}r^2 (\Phi'' + \Phi'^2) - \frac{1}{72}\Phi'r'r + \frac{1}{72}r'^2 - \frac{1}{72}r''r. \quad (31)$$

Here $L \sim 1$; the IR cut off Λ is fixed only by experiment. The terms of higher orders in the derivatives not taken into account in the WKB expansion here can be really omitted if the derivatives of Φ, r are small as compared to unity in the scale of r .

3.Discussion. The expression for the renormalised neutrino stress-energy tensor found differs from the electromagnetic one [10] only by the absolute value of the numerical coefficients; their signs are the same. Also the difference between the radial pressure $\tau = -T_{\rho}^{\rho, \text{ren}}$ and the energy density $\varrho = -T_t^{t, \text{ren}}$ at the wormhole throat (where $r', \Phi' = 0$ and $r'' > 0$) is positive. That is, local weak energy condition *at the throat* is violated. Besides that, integrating $\tau - \varrho$ from $\rho = \rho_0 < 0$ to $\rho = +\infty$ yields the sum of the two explicitly positive (in the wormhole topology $r'(\rho_0) < 0$) terms:

$$\int_{\rho_0}^{\infty} (\tau - \varrho) \exp(-\Phi) d\rho = -\frac{1}{288\pi^2} \frac{r'}{r^3} \exp(-\Phi) \Big|_{\rho=\rho_0} + \frac{1}{96\pi^2} \int_{\rho_0}^{\infty} \frac{r'^2}{r^4} \exp(-\Phi) d\rho > 0. \quad (32)$$

That is, averaged weak energy condition [2] is violated as well. The difference from the electromagnetic case is in the extent of the weak energy conditions violation which is now $16 \div 20$ times weaker.

The estimate for the wormhole size if logarithm is large compared to unity [10] in the presence of N_1 spin 1 and $N_{1/2}$ spin 1/2 massless fields modifies as

$$r_0^2 \simeq \frac{G}{120\pi} (4N_1 + N_{1/2}) \ln \left(\frac{120\pi}{G} \frac{\Lambda^2}{4N_1 + N_{1/2}} \right). \quad (33)$$

Typical size of the wormhole in the neutrino vacuum would be 2 times smaller than that in the electromagnetic vacuum.

It is interesting to compare our expressions for the stress-energy of spin 1/2, 1 massless fields with analogous expression for the massless conformal scalar field. As given in refs. [5, 4], this expression does not contain the terms of the second order in the derivatives of Φ, r . This seems quite natural because anomalous trace which signals that the stress-energy is nonzero in the curved background proves to contain no second order terms just in the conformal scalar case. Therefore in our framework we would get $\tau - \varrho = 0$ (the leading terms with no derivatives should vanish in $\tau - \varrho$ because we must have $\tau = \varrho$ at $\Phi, r = \text{const}$ due to the $t \leftrightarrow \rho$ symmetry in this case) and were to expand to the fourth order derivatives whose values are not restricted by the wormhole topology. Thus, violation of the weak energy conditions in the wormhole topology with slowly varied metric for the scalar vacuum does not take place *automatically* but should follow from the solution of backreaction problem. This is the feature which distinguishes the conformal scalar field from the nonzero spin fields.

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